Worksheet P2 and P4 Derivation IGCSE Mathematics 0580

[Q1 to Q12]

1. For each of the following find $\frac{dy}{dx}$

   (a) $y = x^4$ (b) $y = x^9$ (c) $y = 4x^3$ (d) $y = -4x^{-4}$ (e) $y = -16x^5$ (f) $y = \frac{1}{3}x^3$ (g) $y = \frac{2}{9}x^{-3}$

   (h) $y = 7x^6 + 9x^2$ (i) $y = -7x^2 + 3x^6$ (j) $y = 4x^2 - 12x^3 + 5x^4$

   (k) $y = -\frac{8}{11}x^4 + \frac{2}{7}x^3 - \frac{3}{4}x^2$

2. Find the gradient of each of the following curves at the point or x coordinate indicated.

   (a) $y = x^3$ at point (1,1) (b) $y = -3x^3$ at point (2,-24) (c) $y = -5x^6$ at the point (-2,-320)

   (d) $y = 3x^4 + 6x^3 - 3x^2$ at $x = -1$

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3. Find the co-ordinates of the point at which the curve with equation $y = 2x^3 + 3x^2$ has gradient 12.

4. Find the co-ordinates of the point at which the curve with equation $y = \frac{1}{4}x^4 - \frac{3}{2}x^2$ has gradient 0.

5. Differentiate each of the following

   (a) $y = 4$ (b) $y = 5x$ (c) $y = -4x$ (d) $y = 7x - 6$ (e) $y = -3x + 6$ (f) $y = \frac{1}{3}t^3 + \frac{1}{2}t^2 + \frac{1}{4}$

   (g) $y = mx + c$

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6. Differentiate each of the following

(a) $y = x^3(x^2 + 2x)$ (b) $\frac{1}{5}x^2(x + 3)$ (c) $y = (x+3)^2$ (d) $y = (3x - 2)^2$ (e) $y = (3x - 5)(7x - 3)$

(f) $y = 5(x + 3)(x - 7)$ (g) $y = \frac{2}{3}t^6(t + \frac{1}{4})$ (h) $y = \frac{6x^3+2x^2}{2x}$ (i) $y = \frac{x^3+2x^2}{4x}$

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7. Find the gradient of the curve with equation $y = (3x - 2)(4x + 1)$at the point with co-ordinates (3 , 91)

8. Find the point at which the gradient of the curve with equation $y = 3x^2 - 4x + 1$ is zero.

9. Find the coordinates of the point on the curve with the equation $y = 3x^2 - 4x - 2$ where the tangent is parallel to the line with equation $y = -2x + 1$.

10. Find the coordinates of both points on the curve with equation $y = 2x^3 - 9x^2 + 12x$ where the curve is parallel to x-axis.

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11. The curve with the equation $y = x^3 + 3$ has two tangents parallel to the line with equation $y = 12x - 1$. Find the co-ordinates of the two points.

12. The curve with equation $y = ax^3 - 4x + 1$ is parallel to the line $y = 50x - 1$ at the point with x coordinate 3.
    (a) Find the gradient of the curve at the point with x-coordinate 4.
    (b) Show that the tangent at the point with x-coordinate -3 is also parallel to the same line.

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13. d the equation of the tangent of each curve at the given point.

(a) $y = x^2$ at the point with co-ordinates (3,9)

(b) $y = x^2$ at the point with x co-ordinates -2

(c) $y = x^3 + x^2$ at the point with x co-ordinates 4

(d) $y = 3x^3 - 2x + 1$ at the point with x co-ordinates 1.5

(e) $y = \frac{1}{4}x^2 + \frac{1}{5}x$ at the point with x co-ordinates ½

[Q13 to Q19]

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14. A curve has equation $y = 2x^2 + 3x - 2$. The tangent to the curve at $x = 4$ meets the x axis at the point A. Find the coordinates of the point A.

15. A curve has equation $y = -3x^3 + x - 4$. The tangent to this curve at the point where $x = 1$ meets the x-axis at the point A and y-axis at the point B. Find the area of triangle OAB.

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16. The tangents to the curve with equation $y = x^3 - 3x$ at the point A and B with x co-ordinates -1 and 4 respectively meet at point C. Find the co-ordinates of the point C.

17. Find the turning points on point on each of the following curves and state weather each point is a maximum or minimum.
    (a) $y = x^2 - 4x + 1$ (b) $y = x^2 + 6x - 4$ (c) $y = -x^2 + 8x - 2$
    (d) $y = x^3 - 12x - 1$ (e) $y = -x^3 + 6x^2 + 3$ (f) $y = (x - 5)(x + 5)$
    (g) $y = x\,(2x^2 - 21x + 72)$ (h) $y = x^2(3 - x)$

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18. The height, h meters, of a ball above the ground is given by the formula $h = 7t - 5t^2$ at time t seconds after the ball is thrown upwards.

    (a) Find $\frac{dh}{dt}$ (b) Find the greatest height of the ball above the ground.

19. The population of bacteria in a pond is p thousand, d days after the pond is filled. It is found that $p = d^3 - 12d^2 + 45d$. Find the highest population of bacteria in the pond in the first 4 days after it is filled.

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Answers

1) (a)$4x^3$ (b)$9x^8$ (c)$12x^2$ (d)$16x^{-5}$ (e) - $80x^4$ (f)x^2 (g) - $\frac{2}{3}x^{-4}$ (h)$42x^5 + 18x$ (i)$-14x + 18\ x^5$ (j)$8x - 36x^2 + 20x^3$ (k) - $\frac{32}{11}x^3 + \frac{6}{7}x^2 - \frac{3}{2}x$
2) (a)3 (b)-36 (c)960 (d)12
3) (1,5) and (-2,-4)
4) (0,0) or $(\sqrt{3}, -\frac{9}{4})$ or $(-\sqrt{3}, -\frac{9}{4})$
5) (a) 0 (b) 5 (c)-4 (d) 7 (e) -3 (f) $t^2 + t$ (g)m
6) (a) $5x^4 + 8x^3$ (b)$\frac{3}{5}x^2 + \frac{6}{5}x$ (c) $2x + 6$ (d)$18x - 12$ (e)$42x - 44$ (f)$10x - 20$ (g) $\frac{14}{3}t^6 + t^5$ (h)$6x + 1$ (i) $\frac{x}{2} + \frac{1}{2}$
7) 67
8) (2/3 , -1/3)
9) (1/3 , -3)
10) (1,5) and (2,4)
11) (2,11) and (-2,-5)
12) (a) 92. (b) $\frac{dy}{dx}$ at $x = -3$ is 50
13) (a)$y = 6x - 9$ (b)$y = -4x - 4$ (c)$y = 56x - 144$ (d)$y = 18.25\ x - 19.25$ (e)$y = \frac{9}{20}x - \frac{1}{16}$
14) (34/19 , 0)
15) 1/4
16) (26/9 , 2)
17) (a)(2,-3)min (b)(-3,-13)min (c)(4,14)max (d)(-2,15)max and (2,-17)min (e)(0,3)min and (4,35)max (f)(0,-25)min (g)(3,81)max and (4,80)min (h)(0,0)min and (2,4)max
18) (a)7-10t (b)2.45m
19) 54 thousand

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